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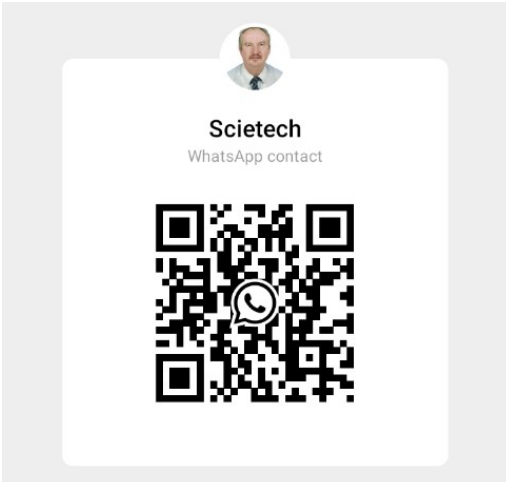


Applied Mathematics for B.Eng. students

Statics (TWB124)

Scietech Teaching: http://scietech.tripod.com

Module:

Vectors; forces; sum of forces at a point; direction cosines and direction angles; components and component vectors.

Introduction to Direction Angles

Cartesian Vectors

The operations of vector algebra, when applied to solving problems in *three dimensions*, are greatly simplified if the vectors are first represented in Cartesian vector form. In this section we will present a general method for doing this; then in the next section we will use this method for finding the resultant force of a system of concurrent forces.

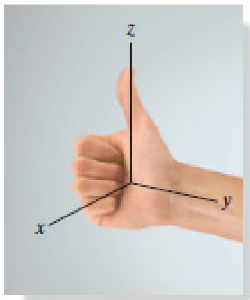


Fig. 2–21

Right-Handed Coordinate System. We will use a right-handed coordinate system to develop the theory of vector algebra that follows. A rectangular coordinate system is said to be *right-handed* if the thumb of the right hand points in the direction of the positive z axis when the right-hand fingers are curled about this axis and directed from the positive x towards the positive y axis, Fig. 2–21.

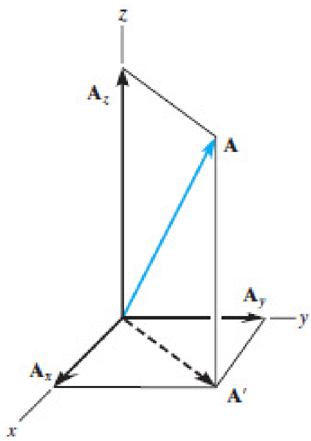


Fig. 2–22

Rectangular Components of a Vector. A vector $\mathbf{A}$ may have one, two, or three rectangular components along the x, y, z coordinate axes, depending on how the vector is oriented relative to the axes. In general, though, when $\mathbf{A}$ is directed within an octant of the x, y, z frame, Fig. 2–22, then by two successive applications of the parallelogram law, we may resolve the vector into components as $\mathbf{A} = \mathbf{A}' + \mathbf{A}_z$ and then $\mathbf{A}' = \mathbf{A}_x + \mathbf{A}_y$. Combining these equations, to eliminate $\mathbf{A}'$, $\mathbf{A}$ is represented by the vector sum of its *three* rectangular components,

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z \tag{2–2}$$

Cartesian Unit Vectors. In three dimensions, the set of Cartesian unit vectors, $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, is used to designate the directions of the x, y, z axes, respectively. As stated in Sec. 2.4, the *sense* (or arrowhead) of these vectors will be represented analytically by a plus or minus sign, depending on whether they are directed along the positive or negative x, y, or z axes. The positive Cartesian unit vectors are shown in Fig. 2–23.

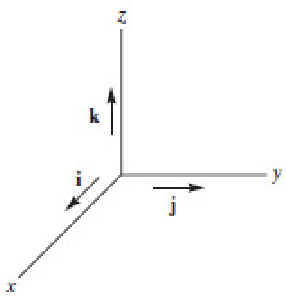


Fig. 2–23

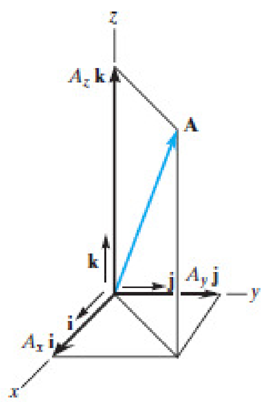


Fig. 2–24

Cartesian Vector Representation. Since the three components of **A** in Eq. 2–2 act in the positive **i**, **j**, and **k** directions, Fig. 2–24, we can write **A** in Cartesian vector form as

$$\boxed{\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}} \tag{2–3}$$

There is a distinct advantage to writing vectors in this manner. Separating the *magnitude* and *direction* of each *component vector* will simplify the operations of vector algebra, particularly in three dimensions.

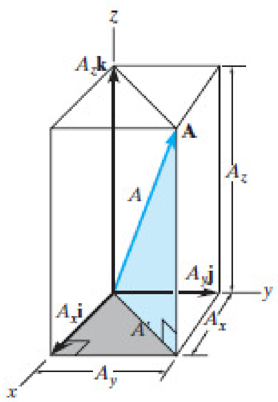


Fig. 2–25

Magnitude of a Cartesian Vector. It is always possible to obtain the magnitude of **A** provided it is expressed in Cartesian vector form. As shown in Fig. 2–25, from the blue right triangle, $A = \sqrt{A'^2 + A_z^2}$, and from the gray right triangle, $A' = \sqrt{A_x^2 + A_y^2}$. Combining these equations to eliminate A' yields

$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}} \tag{2–4}$$

Hence, the magnitude of **A** *is equal to the positive square root of the sum of the squares of its components.*

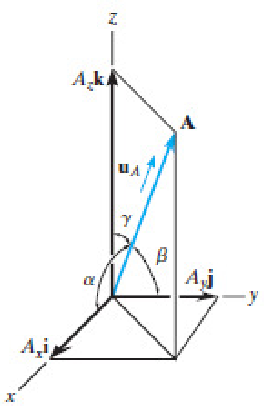


Fig. 2–26

Direction of a Cartesian Vector. We will define the *direction* of **A** by the *coordinate direction angles* α (alpha), β (beta), and γ (gamma), measured between the *tail* of **A** and the *positive* x, y, z axes provided they are located at the tail of **A**, Fig. 2–26. Note that regardless of where **A** is directed, each of these angles will be between 0° and 180°.

To determine α, β, and γ, consider the projection of **A** onto the x, y, z axes, Fig. 2–27. Referring to the blue colored right triangles shown in each figure, we have

$$\boxed{\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\ \gamma = \frac{A_z}{A}} \tag{2–5}$$

These numbers are known as the *direction cosines* of **A**. Once they have been obtained, the coordinate direction angles α, β, γ can then be determined from the inverse cosines.

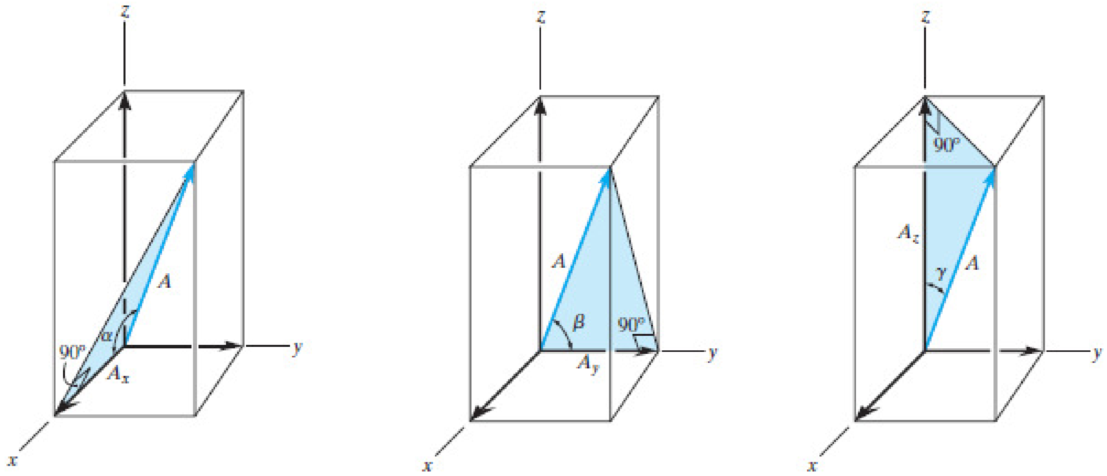


Fig. 2–27

An easy way of obtaining these direction cosines is to form a unit vector $\mathbf{u}_A$ in the direction of A, Fig. 2–26. If $\mathbf{A}$ is expressed in Cartesian vector form, $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$, then $\mathbf{u}_A$ will have a magnitude of one and be dimensionless provided $\mathbf{A}$ is divided by its magnitude, i.e.,

$$\boxed{\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}} \qquad (2\text{–}6)$$

where $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$. By comparison with Eqs. 2–5, it is seen that *the* $\mathbf{i}, \mathbf{j}, \mathbf{k}$ *components of* $\mathbf{u}_A$ *represent the direction cosines of* $\mathbf{A}$, i.e.,

$$\mathbf{u}_A = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k} \qquad (2\text{–}7)$$

Since the magnitude of a vector is equal to the positive square root of the sum of the squares of the magnitudes of its components, and $\mathbf{u}_A$ has a magnitude of one, then from the above equation an important relation among the direction cosines can be formulated as

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1} \qquad (2\text{–}8)$$

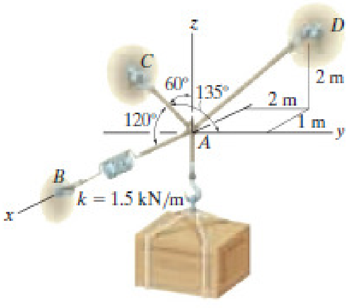
Here we can see that if only *two* of the coordinate angles are known, the third angle can be found using this equation.

Finally, if the magnitude and coordinate direction angles of $\mathbf{A}$ are known, then $\mathbf{A}$ may be expressed in Cartesian vector form as

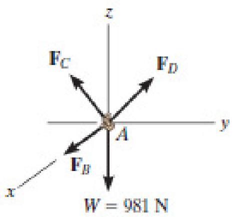
$$\boxed{\begin{aligned}\mathbf{A} &= A\mathbf{u}_A \\ &= A\cos\alpha\,\mathbf{i} + A\cos\beta\,\mathbf{j} + A\cos\gamma\,\mathbf{k} \\ &= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\end{aligned}} \qquad (2\text{–}9)$$

Using Direction Angles

Determine the tension in each cord used to support the 100-kg crate shown in Fig. 3–13*a*.



(a)



(b)

Fig. 3–13

SOLUTION

Free-Body Diagram. The force in each of the cords can be determined by investigating the equilibrium of point A. The free-body diagram is shown in Fig. 3–13*b*. The weight of the crate is $W = 100(9.81) = 981$ N.

Equations of Equilibrium. Each force on the free-body diagram is first expressed in Cartesian vector form. Using Eq. 2–9 for $\mathbf{F}_C$ and noting point $D(-1\text{ m}, 2\text{ m}, 2\text{ m})$ for $\mathbf{F}_D$, we have

$$\mathbf{F}_B = F_B\mathbf{i}$$

$$\mathbf{F}_C = F_C \cos 120°\mathbf{i} + F_C \cos 135°\mathbf{j} + F_C \cos 60°\mathbf{k}$$
$$= -0.5F_C\mathbf{i} - 0.707F_C\mathbf{j} + 0.5F_C\mathbf{k}$$

$$\mathbf{F}_D = F_D\left[\frac{-1\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}}{\sqrt{(-1)^2 + (2)^2 + (2)^2}}\right]$$
$$= -0.333F_D\mathbf{i} + 0.667F_D\mathbf{j} + 0.667F_D\mathbf{k}$$

$$\mathbf{W} = \{-981\mathbf{k}\}\text{ N}$$

Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$
$$F_B\mathbf{i} - 0.5F_C\mathbf{i} - 0.707F_C\mathbf{j} + 0.5F_C\mathbf{k}$$
$$-0.333F_D\mathbf{i} + 0.667F_D\mathbf{j} + 0.667F_D\mathbf{k} - 981\mathbf{k} = \mathbf{0}$$

Equating the respective **i**, **j**, **k** components to zero,

$$\Sigma F_x = 0; \qquad F_B - 0.5F_C - 0.333F_D = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad -0.707F_C + 0.667F_D = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad 0.5F_C + 0.667F_D - 981 = 0 \qquad (3)$$

Solving Eq. (2) for F_D in terms of F_C and substituting this into Eq. (3) yields F_C. F_D is then determined from Eq. (2). Finally, substituting the results into Eq. (1) gives F_B. Hence,

$$F_C = 813\text{ N} \qquad \textit{Ans.}$$
$$F_D = 862\text{ N} \qquad \textit{Ans.}$$
$$F_B = 694\text{ N} \qquad \textit{Ans.}$$

Using Vector Coordinates:

Determine the force in each cable used to support the 40-lb crate shown in Fig. 3–12*a*.

SOLUTION

Free-Body Diagram. As shown in Fig. 3–12*b*, the free-body diagram of point A is considered in order to "expose" the three unknown forces in the cables.

Equations of Equilibrium. First we will express each force in Cartesian vector form. Since the coordinates of points B and C are $B(-3\text{ ft}, -4\text{ ft}, 8\text{ ft})$ and $C(-3\text{ ft}, 4\text{ ft}, 8\text{ ft})$, we have

$$\mathbf{F}_B = F_B\left[\frac{-3\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (8)^2}}\right]$$
$$= -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$
$$\mathbf{F}_C = F_C\left[\frac{-3\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (4)^2 + (8)^2}}\right]$$
$$= -0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k}$$
$$\mathbf{F}_D = F_D\mathbf{i}$$
$$\mathbf{W} = \{-40\mathbf{k}\}\text{ lb}$$

Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$
$$-0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$
$$-0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k} + F_D\mathbf{i} - 40\mathbf{k} = \mathbf{0}$$

Equating the respective **i**, **j**, **k** components to zero yields

$$\Sigma F_x = 0; \qquad -0.318F_B - 0.318F_C + F_D = 0 \qquad (1)$$
$$\Sigma F_y = 0; \qquad -0.424F_B + 0.424F_C = 0 \qquad (2)$$
$$\Sigma F_z = 0; \qquad 0.848F_B + 0.848F_C - 40 = 0 \qquad (3)$$

Equation (2) states that $F_B = F_C$. Thus, solving Eq. (3) for F_B and F_C and substituting the result into Eq. (1) to obtain F_D, we have

$$F_B = F_C = 23.6\text{ lb} \qquad \textit{Ans.}$$
$$F_D = 15.0\text{ lb} \qquad \textit{Ans.}$$

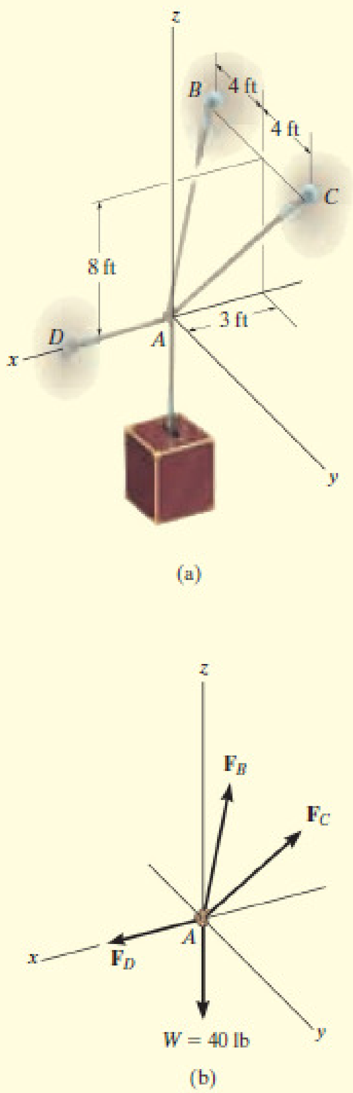
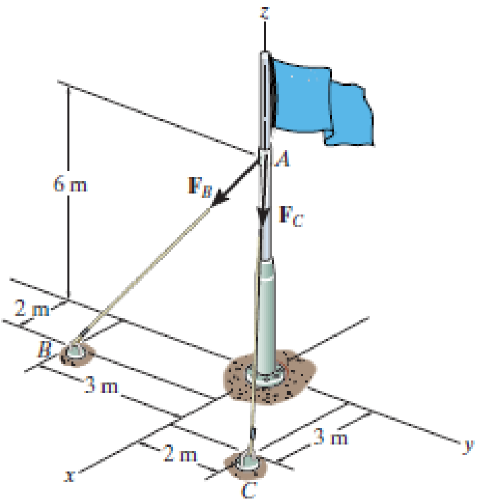


Fig. 3–12

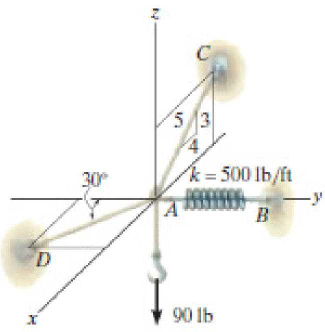
Question

If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.

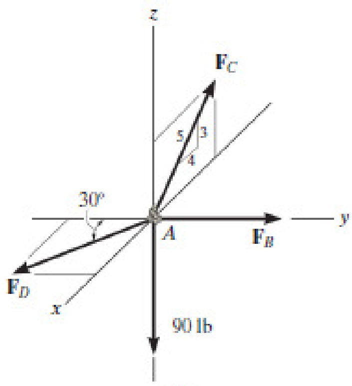


Vector problem

A 90-lb load is suspended from the hook shown in Fig. 3–10*a*. If the load is supported by two cables and a spring having a stiffness $k = 500$ lb/ft, determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x–y plane and cable AC lies in the x–z plane.



(a)



(b)

Fig. 3–10

SOLUTION

The stretch of the spring can be determined once the force in the spring is determined.

Free-Body Diagram. The connection at A is chosen for the equilibrium analysis since the cable forces are concurrent at this point. The free-body diagram is shown in Fig. 3–10*b*.

Equations of Equilibrium. By inspection, each force can easily be resolved into its x, y, z components, and therefore the three scalar equations of equilibrium can be used. Considering components directed along each positive axis as "positive," we have

$$\Sigma F_x = 0; \qquad F_D \sin 30° - \left(\tfrac{4}{5}\right) F_C = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad -F_D \cos 30° + F_B = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad \left(\tfrac{3}{5}\right) F_C - 90 \text{ lb} = 0 \qquad (3)$$

Solving Eq. (3) for F_C, then Eq. (1) for F_D, and finally Eq. (2) for F_B, yields

$$F_C = 150 \text{ lb} \qquad \textit{Ans.}$$

$$F_D = 240 \text{ lb} \qquad \textit{Ans.}$$

$$F_B = 207.8 \text{ lb} = 208 \text{ lb} \qquad \textit{Ans.}$$

The stretch of the spring is therefore

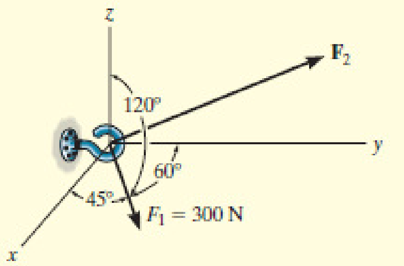
$$F_B = k s_{AB}$$

$$207.8 \text{ lb} = (500 \text{ lb/ft})(s_{AB})$$

$$s_{AB} = 0.416 \text{ ft} \qquad \textit{Ans.}$$

NOTE: Since the results for all the cable forces are positive, each cable is in tension; that is, it pulls on point A as expected, Fig. 3–10*b*.

Direction Angles given



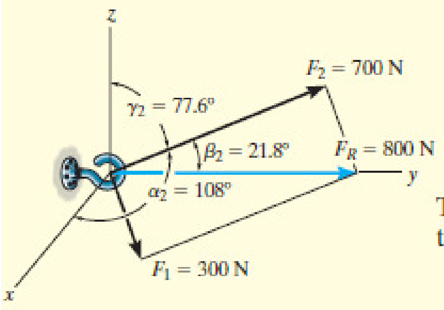


Fig. 2–33

Two forces act on the hook shown in Fig. 2–33*a*. Specify the magnitude of $\mathbf{F}_2$ and its coordinate direction angles so that the resultant force $\mathbf{F}_R$ acts along the positive *y* axis and has a magnitude of 800 N.

SOLUTION

To solve this problem, the resultant force $\mathbf{F}_R$ and its two components, $\mathbf{F}_1$ and $\mathbf{F}_2$, will each be expressed in Cartesian vector form. Then, as shown in Fig. 2–33*b*, it is necessary that $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$.

Applying Eq. 2–9,

$$\begin{aligned}\mathbf{F}_1 &= F_1 \cos \alpha_1 \mathbf{i} + F_1 \cos \beta_1 \mathbf{j} + F_1 \cos \gamma_1 \mathbf{k} \\ &= 300 \cos 45° \,\mathbf{i} + 300 \cos 60° \,\mathbf{j} + 300 \cos 120° \,\mathbf{k} \\ &= \{212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k}\} \text{ N} \\ \mathbf{F}_2 &= F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}\end{aligned}$$

Since $\mathbf{F}_R$ has a magnitude of 800 N and acts in the $+\mathbf{j}$ direction,

$$\mathbf{F}_R = (800 \text{ N})(+\mathbf{j}) = \{800\mathbf{j}\} \text{ N}$$

We require

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\ 800\mathbf{j} &= 212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k} + F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k} \\ 800\mathbf{j} &= (212.1 + F_{2x})\mathbf{i} + (150 + F_{2y})\mathbf{j} + (-150 + F_{2z})\mathbf{k}\end{aligned}$$

To satisfy this equation the **i**, **j**, **k** components of $\mathbf{F}_R$ must be equal to the corresponding **i**, **j**, **k** components of $(\mathbf{F}_1 + \mathbf{F}_2)$. Hence,

$$\begin{aligned}0 &= 212.1 + F_{2x} & F_{2x} &= -212.1 \text{ N} \\ 800 &= 150 + F_{2y} & F_{2y} &= 650 \text{ N} \\ 0 &= -150 + F_{2z} & F_{2z} &= 150 \text{ N}\end{aligned}$$

The magnitude of $\mathbf{F}_2$ is thus

$$\begin{aligned}F_2 &= \sqrt{(-212.1 \text{ N})^2 + (650 \text{ N})^2 + (150 \text{ N})^2} \\ &= 700 \text{ N} \qquad \textit{Ans.}\end{aligned}$$

We can use Eq. 2–9 to determine $\alpha_2, \beta_2, \gamma_2$.

$$\cos \alpha_2 = \frac{-212.1}{700}; \qquad \alpha_2 = 108° \qquad \textit{Ans.}$$

$$\cos \beta_2 = \frac{650}{700}; \qquad \beta_2 = 21.8° \qquad \textit{Ans.}$$

$$\cos \gamma_2 = \frac{150}{700}; \qquad \gamma_2 = 77.6° \qquad \textit{Ans.}$$

These results are shown in Fig. 2–33*b*.

Missing angle problem

Express the force **F** shown in Fig. 2–30 as a Cartesian vector.

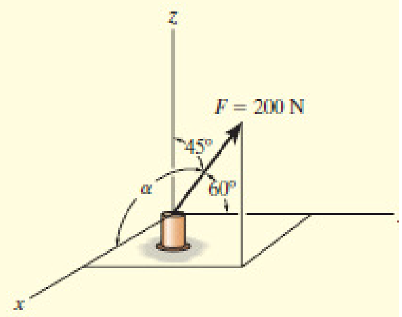


Fig. 2–30

SOLUTION

Since only two coordinate direction angles are specified, the third angle α must be determined from Eq. 2–8; i.e.,

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\cos^2\alpha + \cos^2 60^\circ + \cos^2 45^\circ = 1$$
$$\cos\alpha = \sqrt{1 - (0.5)^2 - (0.707)^2} = \pm 0.5$$

Hence, two possibilities exist, namely,

$$\alpha = \cos^{-1}(0.5) = 60^\circ \qquad \text{or} \qquad \alpha = \cos^{-1}(-0.5) = 120^\circ$$

By inspection it is necessary that $\alpha = 60^\circ$, since $\mathbf{F}_x$ must be in the $+x$ direction.

Using Eq. 2–9, with $F = 200$ N, we have

$$\mathbf{F} = F\cos\alpha\,\mathbf{i} + F\cos\beta\,\mathbf{j} + F\cos\gamma\,\mathbf{k}$$
$$= (200\cos 60^\circ\text{ N})\mathbf{i} + (200\cos 60^\circ\text{ N})\mathbf{j} + (200\cos 45^\circ\text{ N})\mathbf{k}$$
$$= \{100.0\mathbf{i} + 100.0\mathbf{j} + 141.4\mathbf{k}\}\text{ N} \qquad \textit{Ans.}$$

Show that indeed the magnitude of $F = 200$ N.

Determine Angles

Determine the magnitude and the coordinate direction angles of the resultant force acting on the ring in Fig. 2–31*a*.

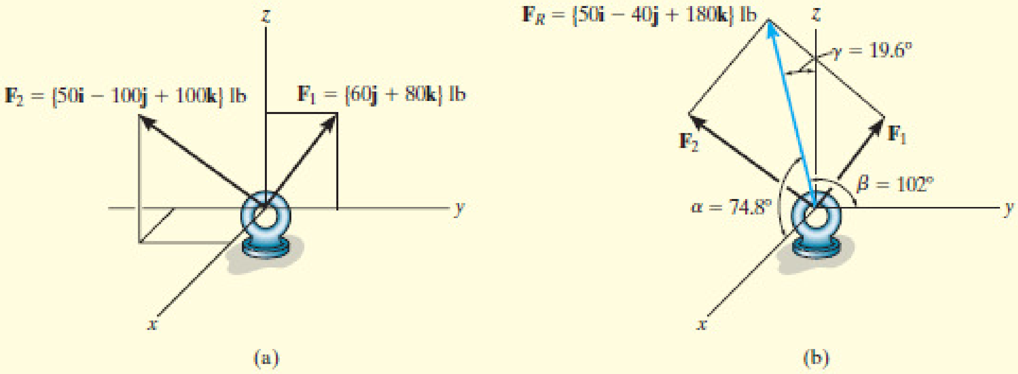


Fig. 2–31

SOLUTION

Since each force is represented in Cartesian vector form, the resultant force, shown in Fig. 2–31*b*, is

$$\mathbf{F}_R = \Sigma\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \{60\mathbf{j} + 80\mathbf{k}\}\text{lb} + \{50\mathbf{i} - 100\mathbf{j} + 100\mathbf{k}\}\text{lb}$$
$$= \{50\mathbf{i} - 40\mathbf{j} + 180\mathbf{k}\}\text{lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(50\text{ lb})^2 + (-40\text{ lb})^2 + (180\text{ lb})^2} = 191.0\text{ lb}$$
$$= 191\text{ lb} \qquad \textit{Ans.}$$

The coordinate direction angles α, β, γ are determined from the components of the unit vector acting in the direction of $\mathbf{F}_R$.

$$\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{50}{191.0}\mathbf{i} - \frac{40}{191.0}\mathbf{j} + \frac{180}{191.0}\mathbf{k}$$
$$= 0.2617\mathbf{i} - 0.2094\mathbf{j} + 0.9422\mathbf{k}$$

so that

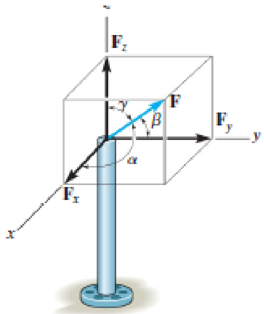
$$\cos\alpha = 0.2617 \qquad \alpha = 74.8° \qquad \textit{Ans.}$$
$$\cos\beta = -0.2094 \qquad \beta = 102° \qquad \textit{Ans.}$$
$$\cos\gamma = 0.9422 \qquad \gamma = 19.6° \qquad \textit{Ans.}$$

These angles are shown in Fig. 2–31*b*.

NOTE: In particular, notice that $\beta > 90°$ since the **j** component of $\mathbf{u}_{F_R}$ is negative. This seems reasonable considering how $\mathbf{F}_1$ and $\mathbf{F}_2$ add according to the parallelogram law.

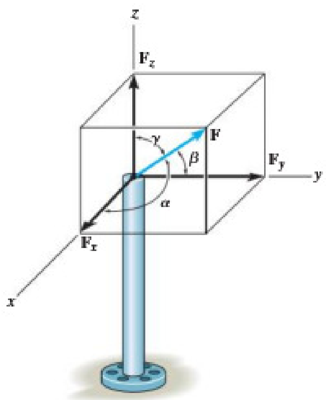
Missing Angle Problem

The pole is subjected to the force **F**, which has components acting along the x, y, z axes as shown. If the magnitude of **F** is 3 kN, $\beta = 30°$, and $\gamma = 75°$, determine the magnitudes of its three components.



***2–84.**

The pole is subjected to the force **F**, which has components acting along the x, y, z axes as shown. If the magnitude of **F** is 3 kN, $\beta = 30°$, and $\gamma = 75°$, determine the magnitudes of its three components.



SOLUTION

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\alpha + \cos^2 30° + \cos^2 75° = 1$$

$$\alpha = 64.67°$$

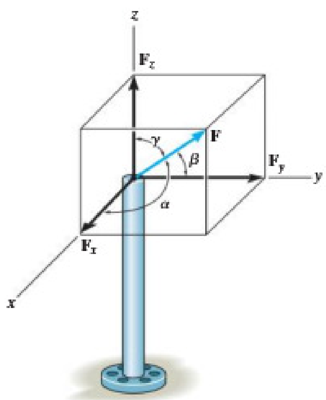
$F_x = 3\cos 64.67° = 1.28\text{ kN}$ **Ans.**

$F_y = 3\cos 30° = 2.60\text{ kN}$ **Ans.**

$F_z = 3\cos 75° = 0.776\text{ kN}$ **Ans.**

2–85.

The pole is subjected to the force **F** which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75°$, determine the magnitudes of **F** and $\mathbf{F}_y$.



SOLUTION

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\left(\frac{1.5}{F}\right)^2 + \cos^2 75° + \left(\frac{1.25}{F}\right)^2 = 1$$

$F = 2.02\text{ kN}$ **Ans.**

$F_y = 2.02\cos 75° = 0.523\text{ kN}$ **Ans.**

Transverse or Azmuth Angles

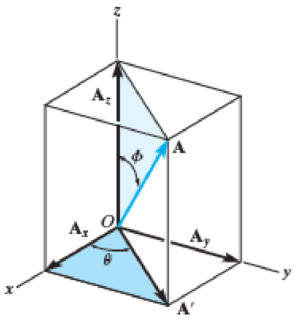


Fig. 2–28

Transverse and Azmuth Angles. Sometimes, the direction of **A** can be specified using two angles, namely, a ***transverse angle*** θ and an ***azmuth angle*** ϕ (phi), such as shown in Fig. 2–28. The components of **A** can then be determined by applying trigonometry first to the light blue right triangle, which yields

$$A_z = A \cos \phi$$

and

$$A' = A \sin \phi$$

Now applying trigonometry to the dark blue right triangle,

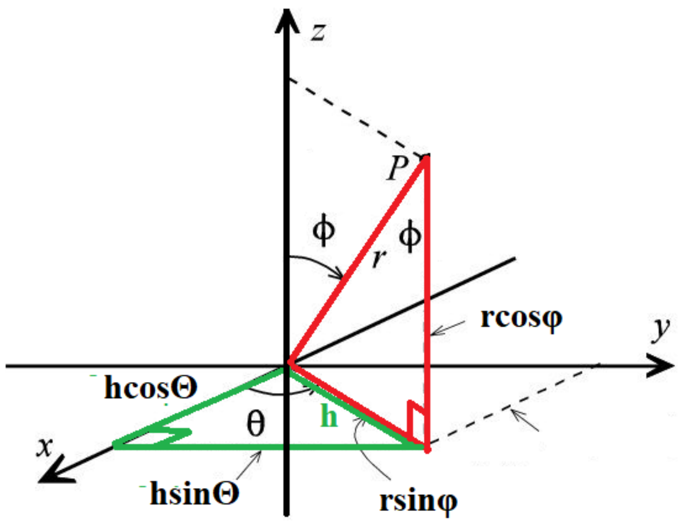
$$A_x = A' \cos \theta = A \sin \phi \cos \theta$$

$$A_y = A' \sin \theta = A \sin \phi \sin \theta$$

Therefore **A** written in Cartesian vector form becomes

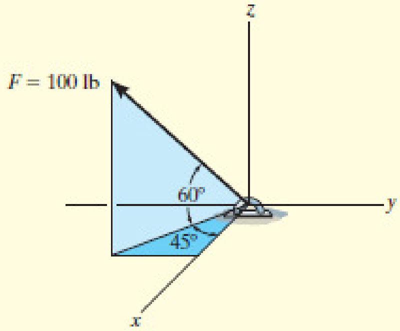
$$\mathbf{A} = A \sin \phi \cos \theta \, \mathbf{i} + A \sin \phi \sin \theta \, \mathbf{j} + A \cos \phi \, \mathbf{k}$$

You should not memorize this equation, rather it is important to understand how the components were determined using trigonometry.



Solving problems with Azmuth angles

Express the force **F** shown in Fig. 2–32*a* as a Cartesian vector.



(a)

SOLUTION

The angles of 60° and 45° defining the direction of **F** are *not* coordinate direction angles. Two successive applications of the parallelogram law are needed to resolve **F** into its x, y, z components. First $\mathbf{F} = \mathbf{F}' + \mathbf{F}_z$, then $\mathbf{F}' = \mathbf{F}_x + \mathbf{F}_y$, Fig. 2–32*b*. By trigonometry, the magnitudes of the components are

$$F_z = 100 \sin 60^\circ \text{ lb} = 86.6 \text{ lb}$$

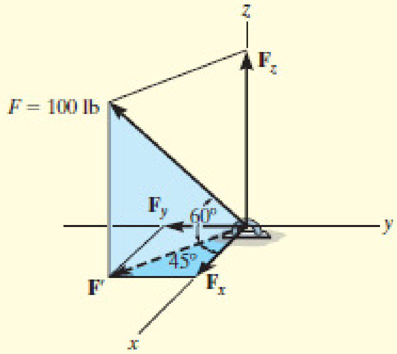
$$F' = 100 \cos 60^\circ \text{ lb} = 50 \text{ lb}$$

$$F_x = F' \cos 45^\circ = 50 \cos 45^\circ \text{ lb} = 35.4 \text{ lb}$$

$$F_y = F' \sin 45^\circ = 50 \sin 45^\circ \text{ lb} = 35.4 \text{ lb}$$

Realizing that $\mathbf{F}_y$ has a direction defined by $-\mathbf{j}$, we have

$$\mathbf{F} = \{35.4\mathbf{i} - 35.4\mathbf{j} + 86.6\mathbf{k}\} \text{ lb} \qquad \textit{Ans.}$$



(b)

To show that the magnitude of this vector is indeed 100 lb, apply Eq. 2–4,

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$= \sqrt{(35.4)^2 + (35.4)^2 + (86.6)^2} = 100 \text{ lb}$$

If needed, the coordinate direction angles of **F** can be determined from the components of the unit vector acting in the direction of **F**. Hence,

$$\mathbf{u} = \frac{\mathbf{F}}{F} = \frac{F_x}{F}\mathbf{i} + \frac{F_y}{F}\mathbf{j} + \frac{F_z}{F}\mathbf{k}$$

$$= \frac{35.4}{100}\mathbf{i} - \frac{35.4}{100}\mathbf{j} + \frac{86.6}{100}\mathbf{k}$$

$$= 0.354\mathbf{i} - 0.354\mathbf{j} + 0.866\mathbf{k}$$

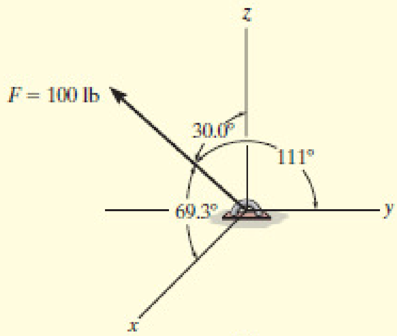
so that

$$\alpha = \cos^{-1}(0.354) = 69.3^\circ$$

$$\beta = \cos^{-1}(-0.354) = 111^\circ$$

$$\gamma = \cos^{-1}(0.866) = 30.0^\circ$$

These results are shown in Fig. 2–32*c*.

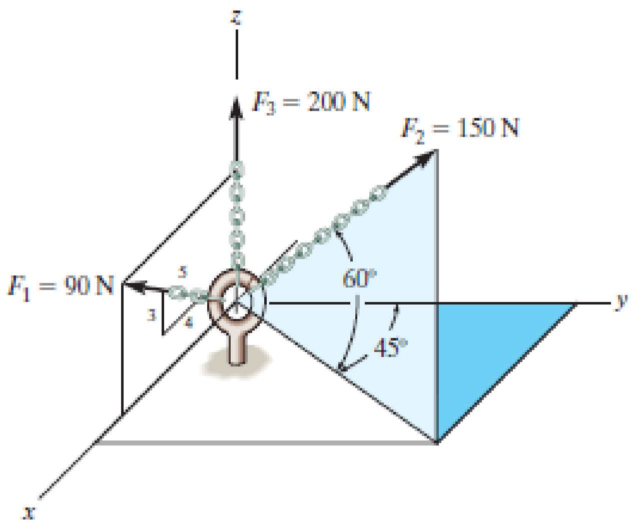


(c)

Fig. 2–32

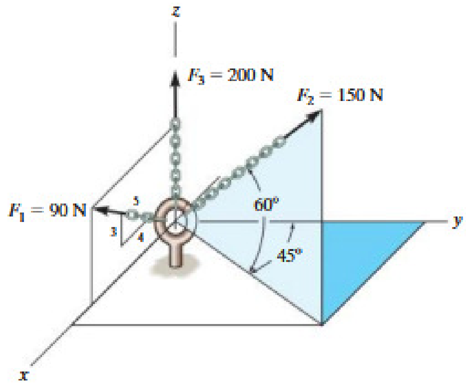
Problem

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



2–74.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$,

$$\mathbf{F}_1 = 90\left(\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right) = \{72.0\mathbf{i} + 54.0\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 150\,(\cos 60°\sin 45°\mathbf{i} + \cos 60°\cos 45°\mathbf{j} + \sin 60°\mathbf{k})$$

$$= \{53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}\}\text{ N}$$

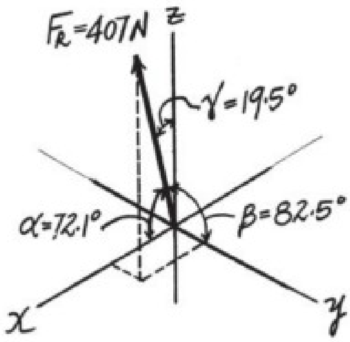
$$\mathbf{F}_3 = \{200\,\mathbf{k}\}\text{ N}$$

Resultant Force.

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (72.0\mathbf{i} + 54.0\mathbf{k}) + (53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}) + (200\mathbf{k})$$

$$= \{125.03\mathbf{i} + 53.03\mathbf{j} + 383.90\}\text{ N}$$



The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{125.03^2 + 53.03^2 + 383.90^2}$$

$$= 407.22\text{ N} = 407\text{ N} \qquad \textbf{Ans.}$$

And the coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{125.03}{407.22}; \qquad \alpha = 72.12° = 72.1° \qquad \textbf{Ans.}$$

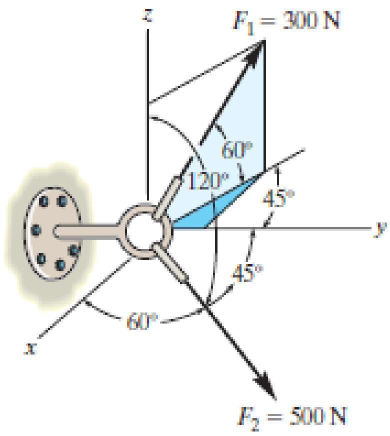
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{53.03}{407.22}; \qquad \beta = 82.52° = 82.5° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{383.90}{407.22}; \qquad \gamma = 19.48° = 19.5° \qquad \textbf{Ans.}$$

Combination of direction and Azmuth angles.

The screw eye is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force. Find the magnitude and coordinate direction angles of the resultant force.

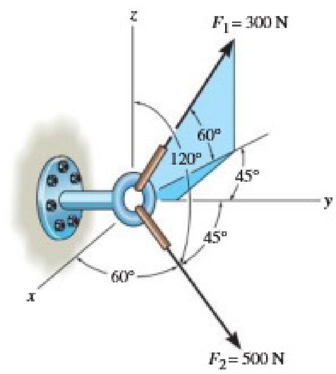
Determine the coordinate direction angles of $\mathbf{F}_1$.



Probs. 2–65/66

2–65.

The screw eye is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force. Find the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$\mathbf{F}_1 = 300(-\cos 60° \sin 45° \mathbf{i} + \cos 60° \cos 45° \mathbf{j} + \sin 60° \mathbf{k})$

$= \{-106.07\mathbf{i} + 106.07\mathbf{j} + 259.81\mathbf{k}\}$ N

$= \{-106\mathbf{i} + 106\mathbf{j} + 260\mathbf{k}\}$ N **Ans.**

$\mathbf{F}_2 = 500(\cos 60° \mathbf{i} + \cos 45° \mathbf{j} + \cos 120° \mathbf{k})$

$= \{250.0\mathbf{i} + 353.55\mathbf{j} - 250.0\mathbf{k}\}$ N

$= \{250\mathbf{i} + 354\mathbf{j} - 250\mathbf{k}\}$ N **Ans.**

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

$= -106.07\mathbf{i} + 106.07\mathbf{j} + 259.81\mathbf{k} + 250.0\mathbf{i} + 353.55\mathbf{j} - 250.0\mathbf{k}$

$= 143.93\mathbf{i} + 459.62\mathbf{j} + 9.81\mathbf{k}$

$= \{144\mathbf{i} + 460\mathbf{j} + 9.81\mathbf{k}\}$ N **Ans.**

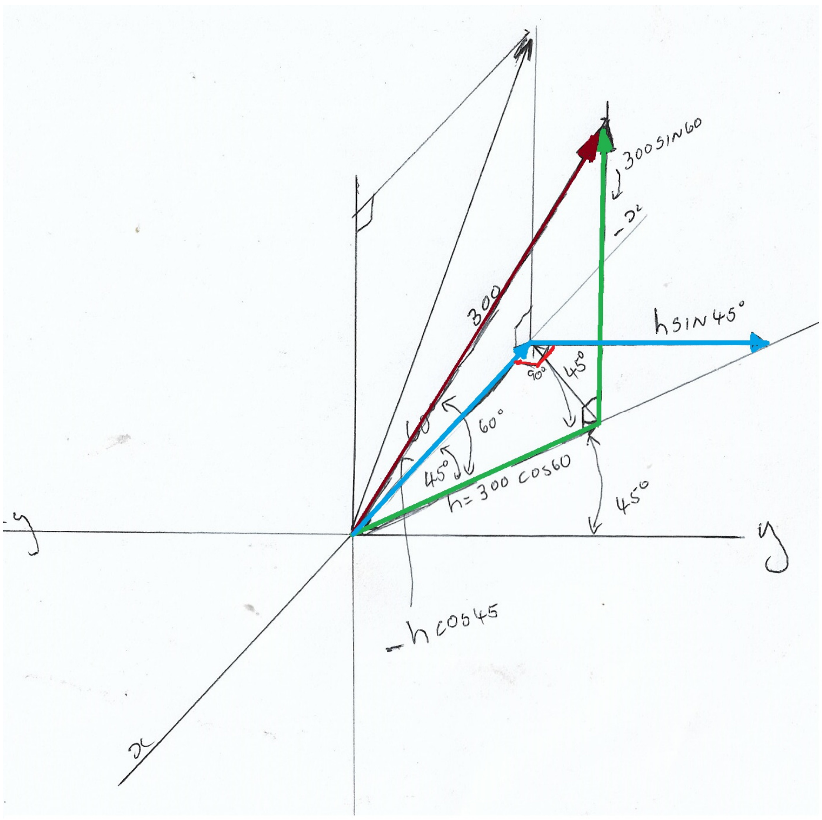
$F_R = \sqrt{143.93^2 + 459.62^2 + 9.81^2} = 481.73 \text{ N} = 482 \text{ N}$ **Ans.**

$$\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{143.93\mathbf{i} + 459.62\mathbf{j} + 9.81\mathbf{k}}{481.73} = 0.2988\mathbf{i} + 0.9541\mathbf{j} + 0.02036\mathbf{k}$$

$\cos\alpha = 0.2988$ $\alpha = 72.6°$ **Ans.**

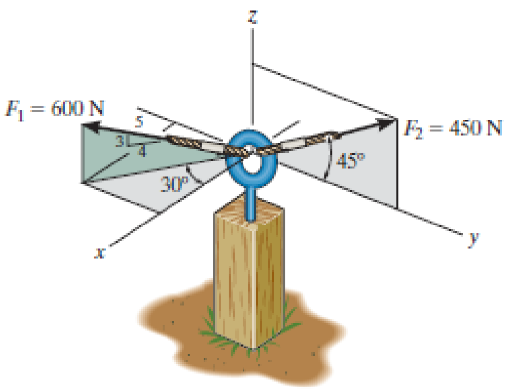
$\cos\beta = 0.9541$ $\beta = 17.4°$ **Ans.**

$\cos\gamma = 0.02036$ $\gamma = 88.8°$ **Ans.**



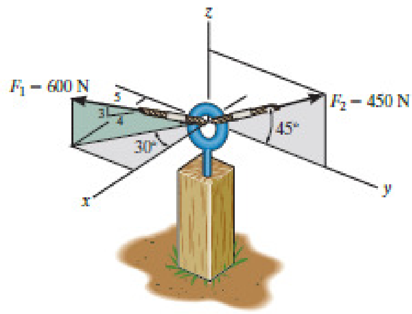
Some more on Azmuth Angles

*2–76. Determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



Probs. 2–75/76

Determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



SOLUTION

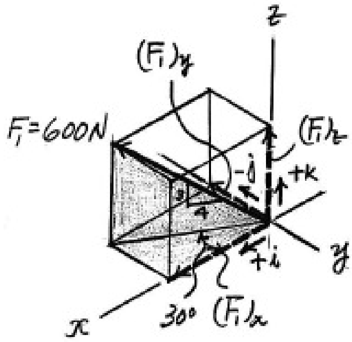
Force Vectors: By resolving $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x, y, and z components, as shown in Figs. a and b, respectively, they are expressed in Cartesian vector form as

$$\mathbf{F}_1 = 600\left(\frac{4}{5}\right)\cos 30°(+\mathbf{i}) + 600\left(\frac{4}{5}\right)\sin 30°(-\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k})$$

$$= \{415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 450\cos 45°(+\mathbf{j}) + 450\sin 45°(+\mathbf{k})$$

$$= \{318.20\mathbf{j} + 318.20\mathbf{k}\}\text{ N}$$



(a)

Resultant Force: The resultant force acting on the eyebolt can be obtained by vectorally adding $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (415.69\mathbf{i} - 240\mathbf{j} + 360\mathbf{k}) + (318.20\mathbf{j} + 318.20\mathbf{k})$$

$$= \{415.69\mathbf{i} + 78.20\mathbf{j} + 678.20\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

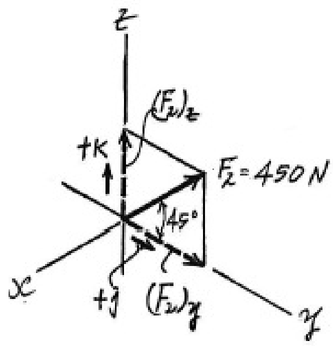
$$= \sqrt{(415.69)^2 + (78.20)^2 + (678.20)^2} = 799.29\text{ N} = 799\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{415.69}{799.29}\right) = 58.7° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{78.20}{799.29}\right) = 84.4° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{678.20}{799.29}\right) = 32.0° \qquad \textbf{Ans.}$$



(b)

Thank you for your support.